Changing Subject Of Formulae

Patrons are reminded that if you get an answer of $x = \frac{a-b}{c-d}$, this is the same as $x = \frac{b-a}{d-c}$; i.e. if you change the sign of *both* the top and bottom of a fraction, then the value of the fraction is unchanged. So if a patron gets $x = \frac{-a-b}{c-d-e}$ as an answer, they may wish to alter it to $x = \frac{a+b}{d+e-c}$ to minimise minus signs.

Change the subject of the following formulae to the letter on the right. If I've written two possible answers, then both are fine (however, on the whole, it is generally preferred to have one fraction rather than the sum or difference of two fractions).

1. $a = x + b$. (x). $\boxed{x = a - b}$

2. $c = d - 2x$. (x). $\boxed{x = \frac{d-c}{2} = \frac{d}{2} - \frac{c}{2}}$

3. $3(x - 5y) = 2(3x + 3y)$. (x). $\boxed{x = -7y}$

4. $3(x - 5y) = 2(3x + 3y)$. (y). $\boxed{y = -\frac{x}{7}}$

5. $x = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. (x_1). $\boxed{x_1 = \frac{xm_1 + xm_2 - m_2x_2}{m_1}}$

6. $s = \left(\dfrac{u+v}{2}\right)t$. (t). $\boxed{t = \frac{2s}{u+v}}$

7. $s = \left(\dfrac{u+v}{2}\right)t$. (v). $\boxed{v = \frac{2s}{t} - u = \frac{2s-ut}{t}}$

8. $V = k\dfrac{p\cos\theta}{r^2}$. (p). $\boxed{p = \frac{Vr^2}{k\cos\theta}}$

9. $V = k\dfrac{p\cos\theta}{r^2}$. (r). $\boxed{r = \pm\sqrt{\frac{kp\cos\theta}{V}}}$

10. $s = ut + \dfrac{1}{2}at^2$. (a). $\boxed{a = \frac{2s-2ut}{t^2}}$

11. $S = 2\pi r^2 + 2\pi rh$. (h). $\boxed{h = \frac{s-2\pi r^2}{2\pi r}}$

12. $V = \dfrac{4}{3}\pi r^3$. (r). $\boxed{r = \sqrt[3]{\frac{3V}{4\pi}}}$

13. $a(x + y) = b(x - 2y)$. (x). $\boxed{x = \frac{ay+2by}{b-a}}$

14. $3(ax - y) = 5(x - ey) + y$. (y). $\boxed{y = \frac{3ax-5x}{4-5e}}$

15. $\dfrac{x-y}{x+y} = m.$ $(x).$ $\boxed{x = \frac{y+my}{1-m}}$

16. $\dfrac{x+y}{2x-3y} = b.$ $(y).$ $\boxed{y = \frac{2bx-x}{1+3b}}$

17. $\sqrt{\dfrac{ax-u}{bx+u}} = c.$ $(x).$ $\boxed{x = \frac{u+c^2u}{a-bc^2}}$

18. $\sqrt[3]{\dfrac{a(x+y)}{b(x-y)}} = d.$ $(x).$ $\boxed{x = \frac{ay+bd^3y}{bd^3-a}}$

19. $x = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}.$ $(m_2).$ $\boxed{m_2 = \frac{xm_1-m_1x_1}{x_2-x}}$

20. $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}.$ $(f).$ $\boxed{f = \frac{d_id_o}{d_i+d_o}}$

21. $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}.$ $(d_i).$ $\boxed{d_i = \frac{fd_o}{d_o-f}}$

22. $\dfrac{1}{f} = \dfrac{1}{d_o} + \dfrac{1}{d_i}.$ $(d_o).$ $\boxed{d_o = \frac{fd_i}{d_i-f}}$

The next few involve completing the square or the quadratic formula. So patrons should only attempt if they've studied this. You have been warned.

23. $x^2 - 4x = 2.$ $(x).$ $\boxed{x = 2 \pm \sqrt{6}}$

24. $5 = u^2 + 12u.$ $(u).$ $\boxed{u = -6 \pm \sqrt{41}}$

25. $z^2 + az = b.$ $(z).$ $\boxed{z = -\frac{a}{2} \pm \sqrt{b + \frac{a^2}{4}}}$

26. $ay^2 = d + cy.$ $(y).$ $\boxed{y = \frac{c}{2a} \pm \sqrt{\frac{c^2}{4a^2} + \frac{d}{a}}}$

27. $s = ut + \dfrac{1}{2}at^2.$ $(t).$ $\boxed{t = -\frac{u}{a} \pm \sqrt{\frac{2s}{a} + \frac{u^2}{a^2}}}$